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GEOPHYSICAL CONTRIBUTIONS IN PRECESSION-NUTATION

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Abstract. Recently we found, from the analysis of Very Long-Baseline Interferometry (VLBI) observations and using resonant effects in several forced nutation terms (Vondrák and Ron 2006a, 2006b, 2007), small quasi-periodic fluctuations of the period of Retrograde Free Core Nutation (RFCN), ranging from 429.8 to 430.8 days. In our preceding studies we were also able to demonstrate that the atmospheric and oceanic excitations are capable of exciting nutation near the resonance of FCN; both amplitude and phase of the geophysically excited pole are consistent with the values observed by VLBI, in the interval of tens of years. The geophysical excitations are now numerically integrated, using Brzezinski's broadband Liouville equations (Brzezinski 1994) in order to estimate the influence of the atmosphere and oceans on precession and nutation. It is then removed from the celestial pole offsets, observed by VLBI. The remaining part is then used to derive the period and quality factor of RFCN. It is shown that the application of real geophysical excitation yields slightly longer period than the MHB sun-synchronous correction.

1. INTRODUCTION

The Earth Orientation Parameters (EOP) describe the full orientation of the Earth in space; they are depicted in Fig. 1. There are five of them:

- Two components of **polar motion** *x, y* that describe the motion of the Earth's spin axis in terrestrial frame. The main components are annual and Chandler wobbles, with periods of 365 and 435 days (with amplitudes smaller than 0.5") 0, plus a secular motion towards Greenland (of about 0.3" per century).
- Almost constant **proper rotation** around the spin axis with roughly diurnal period which is usually expressed in terms of the difference between universal time UT1 and international atomic time TAI, or its time derivative, length-of day. There is a full spectrum of periodic variations, ranging from several days to several decades plus secular deceleration due to tidal friction.

- Two components of precession/nutation that are usually expressed as the **celestial pole offsets** d*X*, d*Y* (the differences between the position of the spin axis in celestial frame from its position predicted by an adopted model of precession-nutation). At present, this model is IAU2000 model of nutation (Mathews et al. 2002) and IAU2006 model of precession (Capitaine et al. 2003). The celestial pole offsets are very small angles, and are mainly caused by Free Core Nutation (see below).

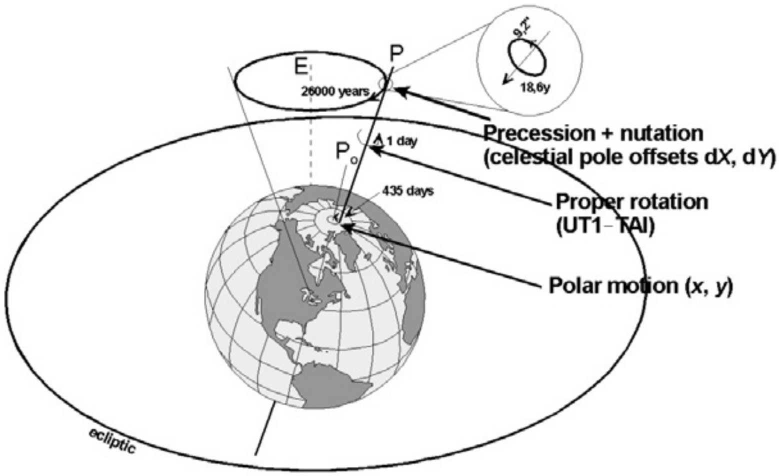

Figure 1: Earth orientation parameters.

The Earth orientation parameters are nowadays determined by four principal observation techniques, whose activities are coordinated by the International Earth Rotation and Reference Systems Service (IERS):

1. Global Positioning System (GPS), which is a satellite navigation system, based on measuring pseudo-ranges between the stations of the surface of the Earth and satellites, emitting the signals. It is principally capable of determining only polar motion and length-of day, in a non-standard solution also the rates of celestial pole offsets.
2. Very Long-Baseline Interferometry (VLBI), which is based on measuring the time delay between the incidence of radio wavefront, coming from extragalactic radio source to two giant antennas (often thousands kilometers apart). This is the only technique capable of determining all five EOP.
3. Satellite and Lunar Laser Ranging (SLR, LLR) that measures the time interval necessary for a short laser pulse to travel from the station to a cubic corn retroreflector located on the satellite or the Moon. It can measure only polar motion and length-of-day.
4. Doppler Orbit determination and Radiopositioning Integrated on Satellite (DORIS), which is a French technique, using Doppler observations of the signals emitted from globally distributed beacons on Earth by receivers,

placed on satellites. This technique is able to determine only polar motion and length-of-day, with somewhat lower precision.

There are two kinds of forces that cause the changes of EOP:

A. External torques, exerted by other solar system bodies – Moon, Sun and planets. These torques have long-periodic character in celestial system, and therefore influence dominantly the nutation.
B. Geophysical fluids (atmosphere, hydrosphere) whose motion has a long-periodic character in terrestrial system, therefore dominantly influencing polar motion. The power at near-diurnal frequencies is very small, but due to strong resonances (see below) the excitations are amplified so that they reach the level of observability.

Important secondary role is also played by deformations of non-rigid Earth, reacting to all these forces. The Earth reacts by slightly changing its figure, and thus its tensor of inertia. The consequence is the modification of amplitudes and phases of all periodic changes. This study, devoted to the problem of resonances and geophysical excitations of nutation, is the continuation of our previous work (Vondrák et al. 2005, Vondrák and Ron 2006a, 2006b, 2007).

2. RESONANCES IN EARTH ROTATION

Due to the existence of a flattened fluid and rigid inner core, there are strong resonances in near-diurnal (in terrestrial frame) part of the spectrum, leading to significant modification of all nutation amplitudes, and also to a non-negligible influence of geophysical excitations in nutation. The strongest of these resonances is the Retrograde Free Core Nutation (RFCN). The situation is depicted in Fig. 2.

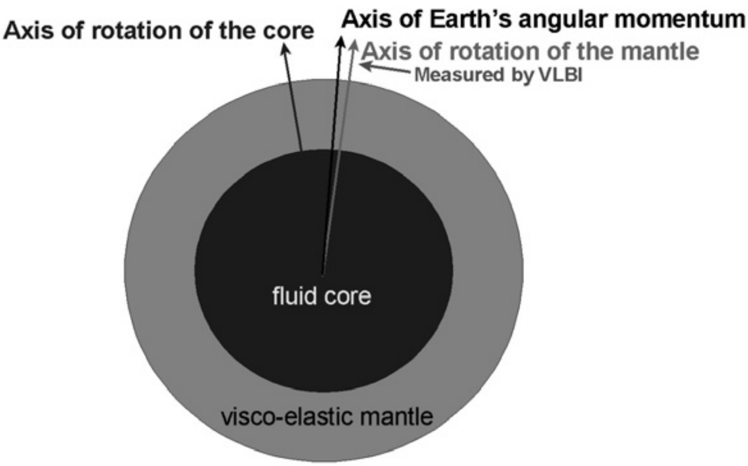

Figure 2: Retrograde Free Core Nutation.

The core can rotate almost independently of the mantle, so the axis of rotation of the fluid core can be slightly inclined to that of the mantle. The observatories, measuring the position of the spin axis with respect to the celestial frame, are located on the surface of the Earth, so they in fact measure the position of the axis of rotation of the mantle. The axis of Earth's angular momentum lies between the two, closer to the axis of a more massive mantle; all three axes lie in one plane. This plane rotates slowly clockwise in space, the period of this motion is about 430 days, and depends dominantly on the dynamical flattening of the core. In the absence of external torque the position of the axis of angular momentum in space remains stable.

All resonances, influencing Earth orientation, are given by MHB transfer function (Mathews et al. 2002), used to derive the IAU2000 model of nutation,

$$T(\sigma) = \frac{e_R - \sigma}{e_R + 1} N_0 \left[1 + (1+\sigma) \left(Q_0 + \sum_{j=1}^{4} \frac{Q_j}{\sigma - s_j} \right) \right]. \quad (1)$$

The complex function $T(\sigma)$ expresses the ratio of the non-rigid amplitude of a forced nutation term with terrestrial frequency σ (in cycles per sidereal day – cpsd) to its rigid Earth value. Here e_R denotes the dynamical ellipticity of the rigid Earth used to compute the rigid solution, N_0, Q_j are complex constants, and s_j are four complex resonance frequencies corresponding to Chandler Wobble (CW, with terrestrial period of about 435 days), RFCN, with celestial period of about 430 days, Prograde Free Core Nutation (PFCN, with celestial period of about 1020 days) and Inner Core Wobble (ICW, with terrestrial period of about 2400 days) respectively. In our case, only $s_2 \approx -1.0023$ cpsd (RFCN frequency) is interesting since it is close to the frequencies of nutation and, at the same time, the corresponding coefficient Q_2 (~4.89×10^{-2}) is almost two hundred times larger than Q_3 (~2.96×10^{-4}), corresponding to PFCN.

The real part of the MHB transfer function (only its small section close to -1 cpsd) is depicted in Fig. 3, together with several closest forced nutation terms. All nutation terms are elliptic and therefore always appear in pairs (describing prograde and retrograde circular motions in celestial frame); they are all retrograde in terrestrial frame, placed symmetrically with respect to –1 cpsd. Closest to RFCN resonance, and therefore the most sensitive to any change of resonance frequency, is evidently the retrograde annual term. PFCN resonance with $s_3 \approx -0.9990$ cpsd is so small that it is invisible in the plot.

MHB transfer function was used to construct the present IAU2000 nutation model; the nutation terms for rigid-Earth, derived by Souchay et al. (1999), were multiplied by MHB transfer function in order to get the non-rigid solution. Then several small additions and corrections, listed by Mathews et al. (2002) in their Tab. 7, were applied. They express the part that is not due to the response of non-rigid Earth to luni-solar and planetary torques. Among them is the "sun-synchronous correction", which in fact is an empirical prograde annual term (with amplitude of about 0.1 mas), removing a discrepancy between the model and VLBI observations at this frequency. It is supposed to account for missing geophysical excitations.

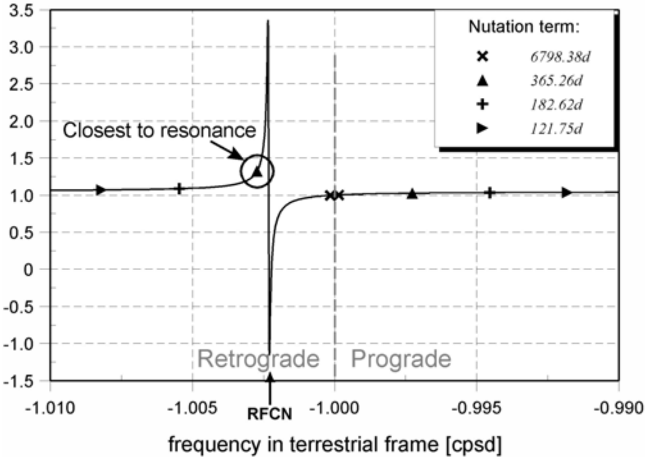

Figure 3: Real part of MHB transfer function.

4. GEOPHYSICALLY EXCITED PART OF NUTATION

In order to get the time series, containing the geophysically excited nutation, a slightly simplified model of Brzezinski (1994), so called broad-band Liouville equations, is used. The equations take into account only the two strongest resonances, i.e., the Chandler wobble with terrestrial frequency $\sigma_C = s_1$, and the Free Core Nutation with terrestrial frequency $\sigma_f = s_2$. The corresponding transfer function in frequency domain, expressing the ratio of the amplitude of the excited pole position P to the amplitude of geophysical excitations χ_p, χ_w (matter and motion term, respectively) with terrestrial frequency σ, is given as

$$P(\sigma) = \chi_p \left[\frac{\sigma_C}{\sigma_C - \sigma} + \frac{9.2\times10^{-2}\sigma_C}{\sigma_f - \sigma} \right] + \chi_w \left[\frac{\sigma_C}{\sigma_C - \sigma} + \frac{5.5\times10^{-4}\sigma_C}{\sigma_f - \sigma} \right]. \tag{2}$$

In case of nutation, the second terms in brackets become dominant, due to the closeness of σ to σ_f (the first terms are important for the excitation of polar motion). The following time series of geophysical excitations (so called angular momentum functions) at 6-hour intervals are used:

- Atmospheric Angular Momentum Functions (pressure + wind terms):
  - NCEP/NCAR re-analysis (Salstein, 2005) in the interval 1983.0 – 2008.0;
  - ERA40 (Thomas et al., 2007, Dobslaw and Thomas, 2007) in the interval 1983.0 – 2001.0.

- Oceanic Angular Momentum Functions (matter + motion terms):
  - ECCO model (Gross et al., 2005) in the interval 1993.0 – 2006.2;
  - OMCT model (Thomas et al., 2007, Dobslaw and Thomas, 2007) in the interval 1983.0 – 2001.0.

These functions are given in terrestrial frame, so they were first transformed into celestial frame (in which nutations are described) and then the periodic signal with periods shorter than 10 days was removed, using the smoothing (Vondrák 1977).

In order to roughly estimate the amplitudes of excited polar motion, we made the FFT spectral analysis of excitations in celestial frame χ' and convoluted the result with Brzezinski transfer function (2). Fig. 4 displays the result, in which we clearly see the dominant FCN term, followed by retrograde and prograde annual terms and somewhat smaller prograde semiannual term.

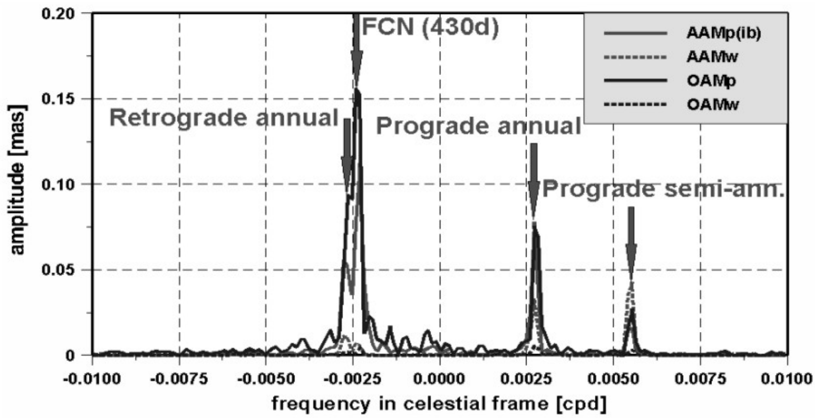

Figure 4: Amplitude spectrum of geophysical excitation, convoluted with Brzezinski transfer function.

This is however only a raw preliminary estimation of what can be expected, with no information about the phases of the geophysical contribution to nutation. To get a more precise estimation in time domain, we use the numerical integration of Brzezinski broad-band Liouville equation in celestial frame and complex form:

$$\ddot{P} - \mathrm{i}(\sigma'_C + \sigma'_f)\dot{P} - \sigma'_C\sigma'_f P = \\ = -\sigma_C\left\{\sigma'_f(\chi'_p + \chi'_w) + \sigma'_C(a_p\chi'_p + a_w\chi'_w) + \mathrm{i}\left[(1+a_p)\dot{\chi}'_p + (1+a_w)\dot{\chi}'_w\right]\right\}, \tag{3}$$

in which

- $P = \mathrm{d}X + \mathrm{i}\mathrm{d}Y$ is the motion of Earth's spin axis in celestial frame due to geophysical excitation;
- $\sigma'_C = 6.32000 + 0.00237\,\mathrm{i}$, $\sigma'_f = -0.0146011 + 0.0001533\,\mathrm{i}$ rad/day are the complex Chandler and FCN frequencies in celestial frame, respectively, whose imaginary parts are closely related to the quality factors;

- $\sigma_C = \sigma'_C - \Omega$ is the Chandler frequency in terrestrial frame, where $\Omega = 6.30038$ rad/day is the angular speed of Earth's rotation;
- χ'_p, χ'_w are excitations (matter and motion term) in celestial frame, and
- $a_p = 9.2\times10^{-2}$, $a_w = 5.5\times10^{-4}$ are dimensionless numerical constants, expressing the response to a matter and motion excitation, respectively.

The numerical integration is made by fourth-order Runge-Kutta method with 6-hour step. Namely we use the procedure `rk4` from Numerical Recipes (Press et al., 1992) that we have adapted to our purpose by rewriting it into complex form. To obtain two first-order equations instead of a second-order one, we use the substitutions $y_1 = P, \quad y_2 = \dot{P} - \mathrm{i}\,\sigma'_C P$, leading to differential equations for two complex functions y_1, y_2

$$\begin{aligned} \dot{y}_1 &= \mathrm{i}\,\sigma'_C y_1 + y_2 \\ \dot{y}_2 &= \mathrm{i}\,\sigma'_f y_2 - \\ &- \sigma_C \left\{ \sigma'_f (\chi'_p + \chi'_w) + \sigma'_C (a_p \chi'_p + a_w \chi'_w) + \mathrm{i}\left[(1 + a_p)\dot{\chi}'_p + (1 + a_w)\dot{\chi}'_w \right] \right\}. \end{aligned} \tag{4}$$

The solution generally yields two free circular motions: prograde Chandler wobble and retrograde FCN with celestial frequencies σ'_C and σ'_f, respectively. Now we need to choose the initial values (in general two complex constants) whose choice defines the amplitudes and phases of both free motions. We are not interested in rapid (nearly diurnal in celestial frame) Chandlerian motion, so we choose only one complex constant, pole position at initial epoch P_0; its first derivative is constrained so that the Chandlerian amplitude disappears. It can be demonstrated that this is assured by choosing the values $y_1(0) = P_0$, $y_2(0) = \mathrm{i}(\sigma'_f - \sigma'_C)P_0$. The final choice of P_0 is then made by repeating the integration with different values P_0 until the fit of the integrated motion to VLBI observations attains a minimum.

The results (four different combinations of atmospheric and oceanic excitations) are shown in Figs. 5 through 8. In all figures, the values of celestial pole offsets (all expressed in milliarcseconds) observed by VLBI are depicted as gray dots, the integrated values by full lines. The root-mean-square (rms) fit between the two series is also given. The first two examples show the excitation by the atmosphere only, with (IB) and without inverted barometer correction, the other two were prepared with the excitation by the atmosphere plus ocean from two different sources. All four examples illustrate relatively good agreement with the observations, both in amplitude and phase, even for several tens of years of integration.

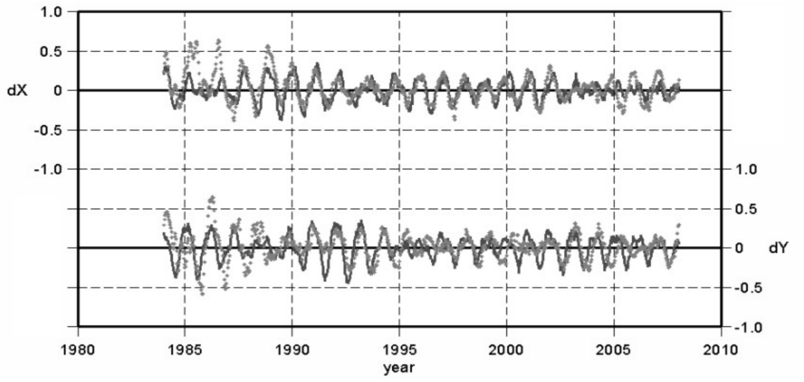

Figure 5: Observed and integrated celestial pole offsets with NCEP(IB). The rms fit is 0.228 mas.

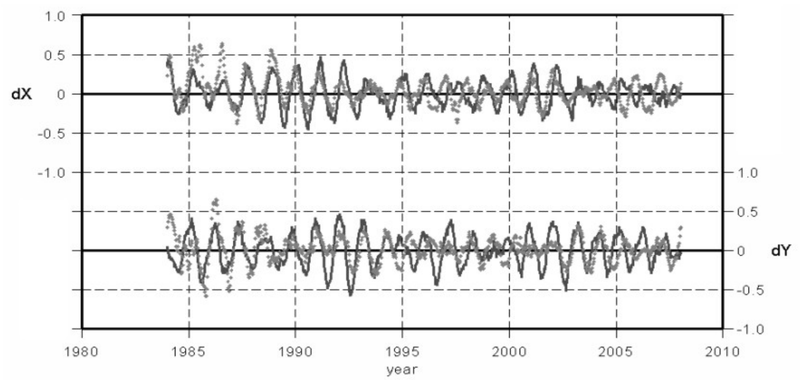

Figure 6: Observed and integrated celestial pole offsets with NCEP. The rms fit is 0.285 mas.

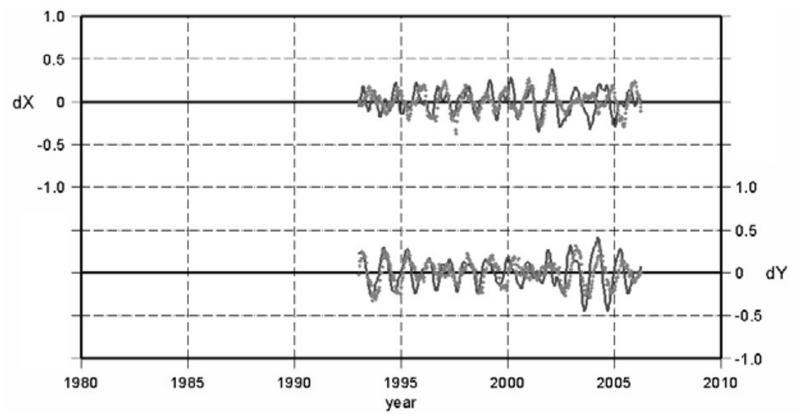

Figure 7: Observed and integrated celestial pole offsets with NCEP(IB) + ECCO. The rms fit is 0.214 mas.

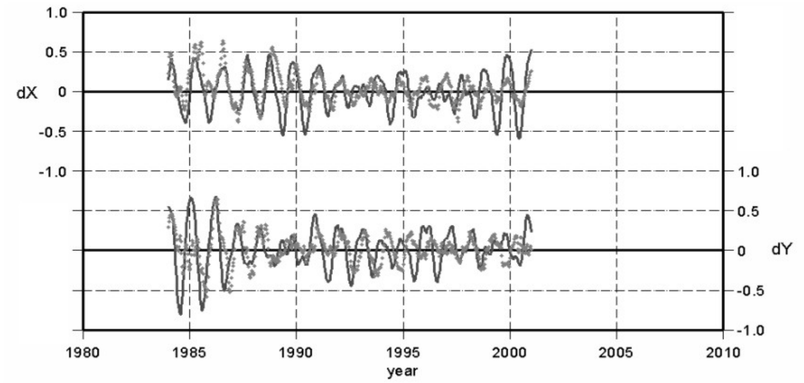

Figure 8: Observed and integrated celestial pole offsets with ERA + OMCT. The rms fit is 0.335 mas.

In the next step we estimated, by applying least-square method to integrated values, the sine/cosine terms of d*X*, d*Y* for the two significant geophysically excited nutation terms, annual and semi-annual, together with RFCN. To derive from them the prograde/retrograde complex amplitudes A^+, A^-, we then used the relation

$$\mathrm{d}X + \mathrm{i}\,\mathrm{d}Y \approx \Delta\psi \sin\varepsilon + \mathrm{i}\,\Delta\varepsilon = -\mathrm{i}\sum_k (A^+ \mathrm{e}^{\mathrm{i}\omega} + A^- \mathrm{e}^{-\mathrm{i}\omega}). \tag{5}$$

The results (in micro-arcseconds) are depicted in Tab. 1, in which also the MHB sun-synchronous correction is given, for comparison. The last column shows the uncertainty of determined values in the respective row. The best agreement with MHB S-S is achieved for only atmospheric excitation with inverted barometer correction, which is also estimated with the smallest uncertainty. On the other hand, the combination ERA+OMCT gives significantly worse results – its agreement both with observations (Fig. 8) and MHB S-S is worse, as well as the uncertainty of the determination of amplitudes.

Table 1. Atmospheric and oceanic contribution to nutation [μas]

	Annual				Semi-annual				
excitation	prograde		retrograde		prograde		retrograde		
AAM+OAM	Re	Im	Re	Im	Re	Im	Re	Im	σ
NCEP(IB)	–3.2	+108.3	–70.5	–25.8	–44.3	–55.5	–0.9	+0.3	±2.6
NCEP	+14.3	+90.8	–77.8	–71.3	–25.3	–51.9	–4.4	–3.2	±4.6
NCEP(IB)+ECCO	–0.7	+110.7	–32.4	–68.4	–46.6	–53.9	–6.3	–21.8	±4.5
ERA+OMCT	–65.7	+180.2	–46.4	+3.2	–17.5	–76.8	+3.2	+7.4	±7.0
MHB S-S	–10.4	+108.2	–	–	–	–	–	–	

5. ESTIMATION OF PERIOD AND QUALITY FACTOR OF RFCN

In our preceding study (Vondrák et al. 2005) we determined the period and quality factor of RFCN by using an indirect method. We first estimated several nutation terms from the observations and removed from them all additions and corrections (including MHB S-S prograde annual term). Then we calculated the values of transfer function at five nutation frequencies and used the MHB transfer function (1) to estimate the complex RFCN frequency.

Now that we have time series of the real geophysically excited nutation, we can use them instead of MHB S-S, using a similar procedure. To this end, we use the combined IVS solution `ivs08q1X.eops` (Schlueter et al. 2002) in interval 1984.0 – 2008.0, from which we removed all MHB Additions and Corrections except the MHB S-S term. Instead of the latter, we removed the integrated geophysical nutation obtained in preceding section. Then we estimated the complex amplitudes (both prograde and retrograde) of the dominant five terms with periods 356.26, 182.62, 121.75, 27.55 and 13.66 days. These were then divided by their rigid-Earth values from (Souchay et al. 1999) to obtain the complex values of transfer function for these frequencies (altogether 20 values) and their standard errors. These, in turn, were used to estimate two parameters of MHB transfer function (1) – $\mathrm{Re}(s_2)$, $\mathrm{Im}(s_2)$. Weighted least-squares estimation was used in successive approximations, all other parameters of Eq. (1) were fixed. Simple relations

$$P = 0.99727/[\mathrm{Re}(s_2)+1], \quad Q = -\mathrm{Re}(s_2)/2\,\mathrm{Im}(s_2) \tag{6}$$

were then used to compute the period P and quality factor Q. The results are shown in Tab. 2, where five different solutions are given (MHB S-S plus the four solutions of Tab. 1). It is evident that the MHB S-S yields the smallest uncertainty, which is quite natural since this correction was derived from the VLBI observations to get the best fit. The other solutions, based on real geophysical excitations, are somewhat less precise and yield systematically longer period.

Table 2. Period and quality factor computed from IVS solution combined with different models of atmosphere/ocean.

Solution IVS +	P	Q
MHB S-S	–430.20 ±0.08	19566 ± 328
NCEP(IB)	–430.91 ±0.24	19716 ± 996
NCEP	–431.16 ±0.19	18430 ± 676
NCEP(IB)+ECCO	–430.91 ±0.24	17606 ± 782
ERA+OMCT	–430.69 ± 0.41	20427 ± 1823

6. CONCLUSIONS

The integration of geophysical excitations by atmosphere and oceans proved to provide time series of celestial pole offsets with a good fit to the values observed by VLBI, in the interval longer than twenty years. Forced nutations due to geophysical excitations are significant for annual and semi-annual terms. These are similar for different models of atmosphere and oceans, prograde annual term is in a good agreement with the empirical MHB sun-synchronous correction. On the other hand, the geophysical contribution to precession seems to be insignificant, as can be seen from Fig. 4, where there is practically no power at zero frequency. The application of real geophysical excitation yields slightly longer period (by less than a day) of RFCN and a little worse fit than the MHB S-S correction.

Acknowledgements

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